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Non-Partisan ‘Get-Out-the-Vote’ Efforts and Policy Outcomes

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ABSTRACT

Non-Partisan ‘Get-Out-the-Vote’ Efforts and Policy Outcomes

by Dan Kovenock and Brian Roberson *

This paper utilizes a simple model of redistributive politics with voter abstention to analyze the impact of nonpartisan ‘get-out-the-vote’ efforts on policy outcomes. Although such efforts are often promoted on the grounds that they provide the social benefit of increasing participation in the electoral process, we find that they have a meaningful impact on policy outcomes and are an important political influence activity for nonprofit advocacy organizations. In equilibrium, nonpartisan gotv efforts are more likely to arise in those segments of the electorate that are sufficiently small and disenfranchised (as measured by the ex ante voter abstention rate). Among those segments in which such efforts arise, the resulting gains are increasing in the level of disenfranchisement of the voters in the segment and decreasing in the segment’s size.

Keywords: *Get out the vote, redistributive politics, nonprofit advocacy organizations, Colonel Blotto game, Tullock game*

JEL Classification: *D72, C72, L30*

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1 Introduction

The National Voter Registration Act of 1993 not only establishes guidelines for governmental agencies, but also specifically encourages nongovernmental entities to take an active role in voter registration. There is a myriad of nonprofit organizations — such as the Association of Community Organizations for Reform Now (ACORN), Declare Yourself, the League of Women Voters, and Rock the Vote to name a few — actively engaged in this effort. However, the tax-exempt status of these nonprofit organizations requires that any get-out-the-vote efforts (henceforth, gotv) be nonpartisan. A natural question that arises is: how do nonpartisan gotv efforts influence policy outcomes?

Although there is extensive research on the effectiveness of the various gotv methods,¹ the theoretical research on how nonpartisan gotv efforts influence policy outcomes is scant.² This is especially true when contrasted to the voluminous research on related political influence activities such as lobbying. One reason for this neglect may be the fact that nonpartisan gotv efforts often claim lofty goals such as promoting democracy.³ However, the most engaged nonprofit organizations are inherently political and are typically attached to a particular segment of the electorate. And, in contrast to encouraging broad participation in government, nonpartisan groups may legally target their gotv efforts in a way that promotes their political objectives.⁴

To examine how targeted nonpartisan gotv efforts influence policy outcomes we utilize

¹See Green and Gerber (2008) for a survey of this research.

²An exception to this is in the sociology literature following Marwell (2004), which considers the role of nonprofit organizations in machine politics.

³This may also partly be due to the fact that in the two-party Hotelling-Downs model voter abstention [as first discussed by Downs 1957 (who uses the term ‘rational’ non-voting) and by Converse 1966 (who uses the term ‘dynamic’ non-voting)] does not alter the policy choices of office-seeking candidates. See for example Hinich and Ordeshook (1969), Ledyard (1984), and Riker and Ordeshook (1973).

⁴According to IRS rules for 501(c)(3)s, nonpartisan gotv efforts may be targeted at groups that are either under represented or that broadly share a set of common interests. For further details see the April 17, 2008 IRS memo which describes the Political Activities Compliance Initiative for the 2008 political campaign season (available at www.irs.gov/pub/irs-tege/2008_paci_program_letter.pdf).

a three-period model of redistributive politics with segmented voters.⁵ The electorate consists of a finite number of disjoint segments, which may differ in size. In each segment an exogenously specified portion (possibly zero) of the citizens abstains from voting, and the abstention rates (or conversely, voter turnout rates) may vary across segments. Each voter prefers higher to lower transfers, and each segment has a nonprofit advocacy organization that represents its interests. In period one, each nonprofit may increase the voter turnout rate in its segment by investing in nonpartisan gotv efforts that target the citizens in its segment. In period two, the two expected vote-share maximizing political parties observe the segments' updated turnout rates, and announce budget-balanced redistributive schedules, which consist of an intra-segment homogeneous transfer level for each segment.⁶ In period three, each of the voters votes for the party that offers the higher transfer.

In equilibrium, only a subset of the nonprofit organizations engage in nonpartisan gotv efforts with the gotv efforts occurring only in those segments of the electorate which are sufficiently small and disenfranchised (as measured by the ex ante abstention rate). In the segments in which nonpartisan gotv efforts arise, the change in the voter turnout rate, as a result of the equilibrium nonpartisan gotv efforts, is increasing in the segment's level of disenfranchisement and is decreasing in the segment's size. The increases in the voter turnout rates in the smaller more disenfranchised segments lead the parties to place relatively greater weight on those segments, which results in higher equilibrium expected transfers to those segments. Conversely, the segments that are larger and more civically engaged (i.e., have lower ex ante abstention rates) receive lower expected transfers.

⁵Our model builds on the redistributive politics literature with segmented voters. See for example Cox and McCubbins (1986), Lindbeck and Weibull (1987), Dixit and Londregan (1995, 1996), Laslier (2002), Strömberg (2004), Schultz (2007), Kovenock and Roberson (2008, 2009), and Roberson (2008) among others.

⁶This may, alternatively, be interpreted as a level of local public good provision under the following assumptions: (1) each citizen has the same preferences for local public good provision, (2) these preferences are linear with respect to the level of local public good provision, (3) there are constant returns to the production of the local public goods, and (4) there are no externalities or spillovers from local public good provision.

The intuition for the equilibrium pattern of nonpartisan gotv efforts and the resulting impact on policy outcomes follows from the interaction of size and disenfranchisement effects. The nonprofit advocacy organizations optimally engage in nonpartisan gotv efforts in order to maximize their respective segment's equilibrium expected transfer — which is strictly increasing in the segment's turnout rate — net of the cost of their nonpartisan gotv efforts. Since each segment's equilibrium expected transfer depends on the segment's turnout rate, the nonprofit advocacy groups in the smaller segments have a size advantage. To increase a segment's turnout rate by any given percentage the number of initially non-voting citizens who must become voting citizens and, hence, the cost of the nonpartisan gotv efforts needed to induce this change, is increasing in the size of the segment. For example consider two segments (A and B), each with a turnout rate of 50%, but in segment A there are 4 citizens (2 voters and 2 non-voters) while in segment B there are 8 citizens (4 voters and 4 non-voters). In order to increase the turnout rate to 75%, segment A needs only one non-voting citizen to become a voter, but segment B needs two additional voters. To summarize, the smaller the segment the larger the marginal effect that each initially non-voting citizen who, through nonpartisan gotv efforts, switches and becomes a voting citizen has on the turnout rate.

In addition to the size effect, in the more disenfranchised segments increasing the turnout rate by any given percentage requires that a lower proportion of initially non-voting citizens become voters as a result of the nonpartisan gotv efforts. If the marginal return to nonpartisan gotv efforts is increasing with respect to the proportion of non-voting citizens, then the nonprofit advocacy groups also encounter a voter disenfranchisement effect. Combining the size and disenfranchisement effects, it follows that the nonprofit advocacy organizations in the smaller more disenfranchised segments can more readily increase their turnout rates and that, as a result, these segments benefit the most from the resulting changes in the equilibrium expected transfers.

Our results indicate that nonpartisan gotv efforts have a meaningful impact on the policy

choices of office-seeking parties, and — in addition to broadly encouraging civic engagement — are an important political influence tool for nonprofit advocacy groups. The competition between nonprofit organizations through nonpartisan gotv efforts is a heretofore unexplored form of special interest politics in which the advocacy efforts are constrained by the regulations on nonprofit organizations. In spite of these constraints, our results are reminiscent of issues that arise in the literature on the combination of lobbying or campaign contributions and electoral competition (see, for example, Austen-Smith 1987, Baron 1994, Besley and Coate 2001, and Grossman and Helpman 1996, 2001). In this setting as in ours, special interest groups simultaneously and non-cooperatively compete in order to influence the outcome of the election and the resulting policies. However, in our setting this competition is over voter turnout and the indirect effect that this has, through the relative weights that the political parties place on the segments, on the election and the resulting policies. Our analysis, thus, extends the literature on special interest politics to allow for nonprofit advocacy groups who use targeted nonpartisan gotv efforts and demonstrates the impact that such efforts have on policy outcomes.

The analysis proceeds as follows. Section 2 presents the multistage model of redistributive politics with targeted nonpartisan gotv efforts. Section 3 characterizes the subgame perfect equilibrium strategies in the model and examines the nature of the equilibrium transfers by the parties and the equilibrium nonpartisan gotv efforts by the nonprofit organizations. Section 4 concludes.

2 The Model

To examine how targeted nonpartisan gotv efforts influence policy outcomes we utilize a three-period model of redistributive politics. In the first (or nonpartisan gotv) stage, each nonprofit organization in each segment of the electorate simultaneously chooses a level of

investment in targeted nonpartisan gotv effort. Within each segment, the targeted nonpartisan gotv efforts increase the turnout rate in that segment. In the second (or campaign) stage, the two political parties observe the segments' updated turnout rates, and each party simultaneously announces a transfer schedule. In the final (or voting) stage, each voter in each segment observes the proposed transfer from each party and votes for the party that offers the higher transfer (with ties broken by fair randomization).

The initial conditions of the game are given as follows. The electorate consists of a finite number n_c of citizens. Each citizen belongs to one of the finite number n_s of identifiable and disjoint segments indexed by $j \in \{1, \dots, n_s\}$. The number of citizens in segment j is denoted m_j , so that $\sum_{j=1}^{n_s} m_j = n_c$. The segments of citizens may be distinguished by characteristics, such as race, gender, age, socioeconomic factors, geographic location, etc.

Citizens may either be voters or non-voters. Within each segment of citizens, a proportion of the citizens abstains from voting in period three.⁷ While we abstract from the exact cause of voter abstention, this may be thought of as arising from considerations such as costly voting. Let $v_j^0 \in (0, 1)$ denote the initial proportion of the citizens in segment j who turn out and vote in the election (henceforth, the *turnout rate*). Alternatively, $1 - v_j^0$ is the initial proportion of the citizens in segment j who abstain from voting (henceforth, the *abstention rate*), and the initial number of non-voters in segment j is $(1 - v_j^0)m_j$. Observe that segment j is distinguished by both its number of citizens m_j and its initial turnout rate v_j^0 . Moreover, j 's share of the total voting population is $m_j v_j^0 / \sum_{h=1}^{n_s} m_h v_h^0$.

⁷In assuming a deterministic and finite number of voters in each segment we are ignoring integer problems that arise when the turnout and abstention rates generate non-whole numbers of citizens who intend to vote or abstain. This integer problem can be avoided without difficulty in a number of ways. First, one can assume, when necessary, the existence of a marginal citizen in the segment that is endowed with a fractional vote rather than a full vote. Alternatively, one can assume that the actual number of citizens voting within a segment is stochastic, but whole, but that the expected turnout and abstention rates generate mathematical expectations that are potentially non-whole. Finally, we can assume the number of citizens in each segment is large and that our continuous treatment is viewed as an arbitrarily close approximation. In fact, our analysis would not be altered substantially if we assumed that each of the finite number of segments contains a continuum of citizens.

Let the three-stage game with segment sizes $\mathbf{m} = (m_1, \dots, m_{n_s})$ and initial turnout rates $\mathbf{v}^0 = (v_1^0, \dots, v_{n_s}^0)$ be denoted $\Gamma(\mathbf{m}, \mathbf{v}^0)$. We start the description of the model in the final stage.

Voting Stage

Suppose that as a result of nonpartisan gotv efforts in the first stage the voter turnout rate in each of the segments in the final voting stage is updated from $\mathbf{v}^0 = (v_1^0, \dots, v_{n_s}^0)$ to $\mathbf{v} = (v_1, \dots, v_{n_s})$. Moreover, let $t_{i,j}$ denote the transfer promised to each citizen in segment j by party $i \in \{A, B\}$ in the second stage of the game. We assume that all citizens prefer higher to lower transfers and in the final stage each citizen that is a voter votes for the party that provides the higher transfer (with ties broken by fair randomization). Hence, if party i provides a strictly higher transfer to segment j than its rival party, it earns the votes of the $m_j v_j$ voters in that segment, with $m_j(1 - v_j)$ citizens abstaining from voting.

Campaign Stage

The second stage consists of a redistributive politics model which extends Laslier (2002) and Laslier and Picard (2002) to allow for voter abstention. At the start of period two, the two expected vote-share maximizing political parties, denoted by $i \in \{A, B\}$, observe the segments' updated turnout rates $\mathbf{v} = (v_1, \dots, v_{n_s})$ and make binding promises as to how they would allocate a fixed budget across the electorate. The fixed budget is normalized to one unit of the homogeneous good. The parties may target campaign promises of different transfers to different segments, but within each segment of citizens, each citizen receives the same transfer. We assume that $t_{i,j}$, the transfer promised to each citizen in segment j by party i , must be nonnegative. For each party, the set of feasible n_s -tuples of transfers across

the n_s segments of the electorate is denoted by

$$\mathcal{T} = \left\{ \mathbf{t} \in \mathbb{R}_+^{n_s} \mid \sum_{j=1}^{n_s} m_j t_j = 1 \right\}.$$

As in Laslier (2002), if a single segment of the electorate contains a majority of the voters, then the unique equilibrium is in pure strategies, in which both of the parties offer the entire budget to the segment with the majority of voters. If no single segment contains a majority of voters, then there are no pure-strategy equilibria in the campaign stage. For each party $i \in \{A, B\}$ a mixed strategy, which we label a *transfer schedule*, is an n_s -variate distribution function $P_i : \mathbb{R}_+^{n_s} \rightarrow [0, 1]$ with support, denoted $\text{Supp}(P_i)$, contained in the set of feasible transfers, \mathcal{T} , and with the set of univariate marginal distribution functions $\{F_{i,j}\}_{j=1}^{n_s}$, one univariate marginal distribution function for each segment of the electorate. The n_s -tuple of party i 's transfer of resources across the n_s segments is a random n_s -tuple drawn from the n_s -variate distribution P_i . Recall that the elements of this random n_s -tuple represent the transfer promised to each citizen in segment j by party i , for $j = 1, \dots, n_s$.

In order to rule out the possibility that the game is initially or becomes degenerate, we assume that no single segment is too large in the sense that if all of the segment's citizens were to turn out and vote the segment would not contain a majority of the voters.

Assumption 1. For all j ,

$$m_j < \sum_{j' \neq j} m_{j'} v_{j'}^0.$$

A direct consequence of Assumption 1 is that neither before nor after the nonpartisan gotv efforts does a single segment contain a majority of the voters.

As is common in the literature on electoral competition, we assume that the *implemented policy* is a probabilistic compromise of the parties' offered transfers, which takes on party A's n_s -tuple of transfers with probability equal to party A's vote share and takes on party

B's n_s -tuple of transfers with probability equal to party B's vote share. Let $E(t_j)$ denote the expected transfer received by each citizen in segment j from the implemented policy generated by the two parties' transfer schedules.

Nonpartisan gotv Stage

In each segment $j \in \{1, \dots, n_s\}$, there is a nonprofit advocacy organization, denoted by NP_j , which represents the segment's interests. In the first stage, each of the nonprofits has the opportunity to make an investment in nonpartisan gotv efforts. The nonpartisan gotv technology works as follows. The nonprofit organization in segment j chooses a target rate $x_j \in [0, 1]$, which represents the proportion of initially non-voting citizens that, as a result of the nonpartisan gotv efforts, become voters. For example, nonpartisan gotv efforts may provide information about voter registration, the location of polling stations, or other information which lowers the cost of voting, and, thereby, increase the turnout rate. Alternatively, the nonpartisan gotv efforts may serve to increase the value of the process benefits accruing from the expressive act of voting. In either case, if the nonprofit chooses a target rate of x_j for the nonpartisan gotv effort, then the proportion of voting citizens increases by $(1 - v_j^0)x_j$. That is, the turnout rate changes from its initial value v_j^0 to the updated value $v_j(x_j)$ as follows

$$v_j(x_j) \equiv v_j^0 + (1 - v_j^0)x_j. \quad (1)$$

After the nonpartisan gotv stage, segment j 's share of the voters is $m_j v_j(x_j) / \sum_{h=1}^{n_s} m_h v_h(x_h)$, which relative to segment j 's initial share of the voters may either increase or decrease depending upon the actions of the nonprofit organizations in the other $n_s - 1$ segments.

Each nonprofit advocacy organization's objective function is assumed to be linearly separable in the costs and benefits of gotv effort. By choosing a target rate of $x_j \in [0, 1]$ for the

nonpartisan gotv effort, the nonprofit organization incurs a cost of $x_j c(m_j, v_j^0)$, where the constant marginal cost $c(m_j, v_j^0)$ satisfies the following assumption.

Assumption 2. *The constant marginal cost of nonpartisan gotv effort is given by*

$$c(m_j, v_j^0) = \alpha m_j (1 - v_j^0)$$

where α is a constant that is greater than $1/(\sum_{h=1}^{n_s} m_h v_h^0)$.

A constant marginal cost of the form given in Assumption 2 corresponds to a constant unit cost per new voter equal to α . That is, increasing the number of voters in segment j by $m_j(1 - v_j^0)x_j$ entails a total cost of $x_j c(m_j, v_j^0) = \alpha m_j(1 - v_j^0)x_j$, and the resulting constant unit cost per voter is $(\alpha m_j(1 - v_j^0)x_j)/(m_j(1 - v_j^0)x_j) = \alpha$. While this is a stylistic assumption, this choice of cost structure is motivated as follows. Given the high level of information that is available to nonprofit organizations and the high degree of targetability in the standard gotv methods (direct mail, phone banks, door-to-door, etc), nonprofit advocacy organizations have the ability to identify the non-voting citizens and to directly target their nonpartisan gotv efforts at the non-voters. It, therefore, seems reasonable to assume a constant unit cost per new voter. However, our main results are qualitatively similar under the assumption that gotv efforts must be broadly targeted at the entire segment rather than at just the non-voters.⁸ Assumption 2's condition on the constant unit cost per new voter α ($\alpha > 1/\sum_{h=1}^{n_s} m_h v_h^0$) rules out the possibility that any nonprofit would optimally choose a target rate that resulted in full participation ($x_j = 1$).

Each of the nonprofit advocacy organizations is risk neutral and seeks to maximize the

⁸It is straightforward to extend Theorem 2 to allow for alternative cost specifications. Under the assumption that the constant marginal cost depends on only the number of citizens (an assumption consistent with either or both imperfect targeting and uncertainty over the identity of non-voters), a slightly stronger form of Propositions 2 and 3 hold as long as the constant marginal cost function $c(m_j)$ is elastic with respect to the number of citizens in the segment. Note that under Assumption 2 the constant marginal cost is unit elastic with respect to the segment size and, hence, forms an endpoint of the set of the elastic marginal cost functions with respect to segment size.

total expected value of the transfers that its segment receives from the implemented policy minus the opportunity cost of the funds invested in nonpartisan gotv efforts,

$$\pi_{NP_j}(x_j, x_{-j}) = m_j E(t_j | x_j, x_{-j}) - x_j c(m_j, v_j^0), \quad (2)$$

where $E(t_j | x_j, x_{-j})$ is the expected transfer that each citizen in segment j expects to receive conditional on the n_s -tuple of nonpartisan gotv efforts \mathbf{x} . Given the normalized budget of one unit of the homogenous good, the total value of the transfers that segment j receives from the implemented policy $m_j E(t_j | x_j, x_{-j})$ is equivalent to, and will henceforth be referred to as, segment j 's expected share of the budget.

3 Results

Since it is individually rational for each voter to vote for the party that offers the higher transfer (doing so increases the expected transfer from the implemented policy), we start our analysis in the campaign stage and work our way back through the game tree. The second stage equilibrium transfer schedules are provided in Theorem 1.

Campaign Stage

Theorem 1. *Let $\mathbf{v} = (v_1, \dots, v_{n_s})$ denote the turnout rates facing the two parties in a subgame starting at the campaign stage of the game. A pair of transfer schedules (P_A^*, P_B^*) constitute a subgame perfect equilibrium pair of local strategies for the subgame starting at \mathbf{v} if and only if the following two conditions are satisfied: (1) $\text{Supp}(P_i^*) \subset \mathcal{T}$ and (2) P_i^* provides the corresponding unique set of univariate marginal distribution functions $\{F_{i,j}^*\}_{j=1}^{n_s}$*

where $\forall j \in \{1, \dots, n_s\}$

$$F_{i,j}^*(t) = \frac{t}{2v_j / \sum_{h=1}^{n_s} v_h m_h} \quad \text{for } t \in \left[0, \frac{2v_j}{\sum_{h=1}^{n_s} v_h m_h}\right].$$

Moreover, such subgame perfect equilibrium local strategies exist and give an expected payoff to each party of $1/2$ of the vote share.

Proof. The existence of a pair of n_s -variate distribution functions which satisfy conditions (1) and (2) of Theorem 1 is provided in the appendix. The proof of the uniqueness of the equilibrium sets of univariate marginal distribution functions is also given in the appendix.

In the following proof we show that any pair of n_s -variate distribution functions which satisfy conditions (1) and (2) of Theorem 1 form an equilibrium. It is sufficient to show that the expected vote share to each party from any budget-balanced strategy is less than or equal to $1/2$, given that the opposition party uses a joint distribution with the univariate marginals outlined above and that expends the budget with probability one.

First note that if the n_s -tuple of initial turnout rates $\{v_j^0\}_{j=1}^{n_s}$ satisfies Assumption 1 [i.e. that $m_j < \sum_{j' \neq j} v_{j'}^0 m_{j'}$ for all j], then it is clear that $v_j m_j < \sum_{j' \neq j} v_{j'} m_{j'}$ for all j , and so, no segment contains a majority of the voters.

Suppose that party A plays an arbitrary budget-balanced mixed strategy \bar{P}_A with the set of univariate marginals $\{\bar{F}_{A,j}\}_{j=1}^{n_s}$. Note that since \bar{P}_A is budget-balanced, it follows that $\text{Supp}(\bar{P}_A) \subset \mathcal{T}$. Also observe that if party B follows an equilibrium strategy P_B^* that satisfies condition (1) and has the unique set of univariate marginals $\{F_{B,j}^*\}_{j=1}^{n_s}$ that satisfy condition (2) outlined above, then $\text{Supp}(P_B^*)$ is contained in the intersection of the n_s -box $\prod_{j=1}^{n_s} [0, 2v_j / (\sum_{h=1}^{n_s} v_h m_h)]$ and the hyperplane \mathcal{T} .

Party A 's expected payoff, $\pi_A(\cdot)$ is calculated as

$$\pi_A(\bar{P}_A, P_B^*) = \frac{1}{\sum_{j=1}^{n_s} v_j m_j} \sum_{j=1}^{n_s} v_j m_j \left(\int_0^\infty F_{B,j}^*(t) d\bar{F}_{A,j}(t) \right) \quad (3)$$

In equation (3), the denominator of the first expression, $\sum_{j=1}^{n_s} v_j m_j$, denotes the number of citizens that vote in the election. While each party maximizes their expected vote share, some of the citizens do not vote in the election, and this subset of citizens is not included in the vote share calculations. In the second term in equation (3), the expression $\sum_{j=1}^{n_s} v_j m_j (\int_0^\infty F_{B,j}^*(t) d\bar{F}_{A,j}(t))$, denotes the expected number of voters to whom party A provides a higher transfer.

Since party B 's transfers, drawn from an equilibrium strategy P_B^* , are contained in the n_s -box $\prod_{j=1}^{n_s} [0, 2v_j / (\sum_{h=1}^{n_s} v_h m_h)]$, it is clear that in any optimal strategy party A never provides transfers outside this n_s -box. Inserting the unique set of equilibrium univariate marginals for party B , $\{F_{B,j}^*\}_{j=1}^{n_s}$, into equation (3) and simplifying yields,

$$\pi_A(\bar{P}_A, P_B^*) = \frac{1}{\sum_{j=1}^{n_s} v_j m_j} \sum_{j=1}^{n_s} v_j m_j \left(\int_0^{\frac{2v_j}{\sum_{h=1}^{n_s} v_h m_h}} \frac{t}{2v_j / \sum_{h=1}^{n_s} v_h m_h} d\bar{F}_{A,j}(t) \right) \quad (4)$$

In any optimal strategy the budget is spent with probability one, and it follows that it is spent in expectation as well, i.e. $\sum_{j=1}^{n_s} m_j \int_0^\infty t d\bar{F}_{A,j} = 1$. Thus, $\pi_A(\bar{P}_A, P_B^*) \leq (1/2)$ since $\text{Supp}(\bar{P}_A) \subset \mathcal{T}$. If in addition $\text{Supp}(\bar{P}_A)$ is contained in the set $\prod_{j=1}^{n_s} [0, 2v_j / (\sum_{j=1}^{n_s} v_j m_j)]$, then $\pi_A(\bar{P}_A, P_B^*) = (1/2)$. This completes the proof that the expected vote share to each party from any budget-balanced strategy is less than or equal to $1/2$, given that the opposition party is using a joint distribution with the univariate marginals outlined above and that expends the budget with probability one. \square

The key feature of both parties' equilibrium transfer schedules, and hence, the implemented policy, is that each segment's expected share of the budget, $m_j E(t_j)$, is identical to its share of the voters $m_j v_j(x_j^*) / \sum_{h=1}^{n_s} m_h v_h(x_h^*)$. As stated in Proposition 1, this feature of the equilibrium expected transfers implies that in each segment j , the expected share of the budget is increasing in its turnout rate v_j .

Proposition 1. *In each segment j , the expected share of the budget $m_j E(t_j)$ is equal to the share of voters $m_j v_j(x_j^*) / \sum_{h=1}^{n_s} m_h v_h(x_h^*)$ which is increasing in the turnout rate $v_j(x_j^*)$.*

Proposition 1 follows directly from the characterization of the equilibrium transfer schedules given in Theorem 1. Note that since each segment's share of the voters, and hence expected budget share, is increasing in its turnout rate, each of the nonprofit advocacy organizations has incentive to engage in nonpartisan gotv efforts. However, in each segment, the share of the voters is also decreasing in the turnout rates of each of the other segments. In the next section we characterize the optimal nonpartisan gotv efforts and examine the resulting changes in the segments' voter turnout rates and expected budget shares.

Nonpartisan gotv Stage

We now solve for the unique subgame perfect equilibrium local strategies in the nonpartisan gotv stage. Recall that in each segment j , if the nonprofit advocacy organization chooses a target rate of $x_j \in [0, 1]$ for the nonpartisan gotv efforts, then the updated turnout rate in segment j , given in equation (1), is $v_j(x_j) = v_j^0 + (1 - v_j^0)x_j$, and the nonprofit incurs a cost of $x_j c(m_j, v_j^0)$. The nonprofit seeks to maximize its expected payoff, given in equation (2), by choosing a target rate for reducing voter abstention. Given the equilibrium expected budget shares (see Proposition 1) the optimization problem for the nonprofit organization in segment j may be written as

$$\max_{x_j \in [0, 1]} \pi_{NP_j}(x_j, x_{-j}) = \max_{x_j \in [0, 1]} \frac{m_j v_j(x_j)}{\sum_{h=1}^{n_s} m_h v_h(x_h)} - x_j c(m_j, v_j^0). \quad (5)$$

Theorem 2 establishes the existence of a unique subgame perfect equilibrium profile of local strategies in the nonpartisan gotv stage. Note that in the nonpartisan gotv stage, the optimization problem in equation (5) is isomorphic to the optimization problem faced by each contestant in an n_s -player Tullock game (Tullock 1980). The proof given here extends

the analysis of the multi-player Tullock game to allow for asymmetric head-start advantages (i.e., the initial number of voters $m_j v_j^0$ in each segment j).

As we will show, some nonprofit organizations may choose not to engage in any nonpartisan gotv efforts. Without loss of generality, number the segments in nondecreasing order with respect to the expression $m_j v_j^0$: $m_1 v_1^0 \leq m_2 v_2^0 \leq \dots \leq m_{n_s} v_{n_s}^0$. Let \mathcal{P} denote the set of indices of the segments in which the nonprofits participate in gotv efforts (i.e., optimally choose strictly positive targets $x_j^* > 0$ for gotv efforts), and let $k^* \leq n_s$ denote the number of segments in which nonprofits choose to participate in gotv efforts. It will also be helpful to define the expression \mathcal{V}_k for $k = 1, \dots, n_s$ as follows,

$$\mathcal{V}_k = \frac{(k-1) + \left[(k-1)^2 + 4(\alpha k) \left(\sum_{j>k} m_j v_j^0 \right) \right]^{1/2}}{2(\alpha k)}.$$

In the event that $k = k^*$, we will show that \mathcal{V}_{k^*} is equal to the equilibrium number of voters $\sum_{j=1}^{n_s} m_j v_j(x_j^*)$.

Theorem 2. *In the nonpartisan gotv stage of the game with n_s -tuples of initial turnout rates $\mathbf{v}^0 = (v_1^0, \dots, v_{n_s}^0)$ and segment sizes $\mathbf{m} = (m_1^0, \dots, m_{n_s}^0)$ that satisfy Assumption 1, there exists a unique pure-strategy subgame perfect equilibrium given by*

$$x_j^* = \begin{cases} \frac{1}{m_j(1-v_j^0)} \left[\mathcal{V}_{k^*} - \alpha \mathcal{V}_{k^*}^2 - m_j v_j^0 \right] & \text{if } j \leq k^* \\ 0 & \text{if } j > k^* \end{cases}$$

where k^* is the largest index k such that

$$\mathcal{V}_{(k-1)} - \alpha \mathcal{V}_{(k-1)}^2 > m_k v_k^0.$$

The equilibrium number of voters is

$$\sum_{j=1}^{n_s} m_j v_j(x_j^*) = \mathcal{V}_{k^*} = \frac{\left(k^* - 1\right) + \left[\left(k^* - 1\right)^2 + 4\left(\alpha k^*\right)\left(\sum_{j>k^*} m_j v_j^0\right)\right]^{1/2}}{2\left(\alpha k^*\right)}.$$

Proof. Given the relationship between the nonpartisan gotv stage and the multi-player Tullock game, the following characterization of equilibrium strategies builds upon the characterization of the multi-player Tullock game by Hillman and Riley (1989), Stein (2002), and Matros (2006).

First note that it is clear that

$$\frac{m_j}{m_j + \sum_{j' \neq j} m_{j'} v_{j'}(x_{j'})} - \frac{m_j v_j^0}{m_j v_j^0 + \sum_{j' \neq j} m_{j'} v_{j'}(x_{j'})} < \frac{m_j (1 - v_j^0)}{\sum_{h=1}^{n_s} m_h v_h^0} \quad (6)$$

for all j . Recall that each nonprofit can choose not to participate in nonpartisan gotv efforts and have a strictly positive payoff. It follows from the nonprofit organization's payoff function, given in equation (5), that for any $(n_s - 1)$ -tuple of gotv efforts $\mathbf{x}_{-j} \in [0, 1]^{(n_s-1)}$ the expected payoff in each segment j from choosing $x_j = 1$ is strictly less than the payoff from choosing $x_j = 0$ if

$$\frac{m_j}{m_j + \sum_{j' \neq j} m_{j'} v_{j'}(x_{j'})} - c(m_j, v_j^0) < \frac{m_j v_j^0}{m_j v_j^0 + \sum_{j' \neq j} m_{j'} v_{j'}(x_{j'})}$$

From Assumption 2, $c(m_j, v_j^0) = \alpha m_j (1 - v_j^0) > (m_j (1 - v_j^0) / \sum_{h=1}^{n_s} m_h v_h^0)$. Combining this with equation (6) it is clearly suboptimal for any nonprofit to set $x_j = 1$, and, thus, the relevant portion of the strategy space is $\mathbf{x} \in [0, 1]^{n_s}$.

At an interior solution the first-order condition for the nonprofit's optimization problem

(see equation 5) is

$$\frac{m_j(1 - v_j^0) \left(\sum_{j' \neq j} m_{j'} v_{j'}(x_{j'}) \right)}{\left(\sum_{h=1}^{n_s} m_h v_h(x_h) \right)^2} - \alpha m_j(1 - v_j^0) = 0 \quad (7)$$

The second-order condition for this optimization problem is

$$-\frac{2m_j^2(1 - v_j^0)^2 \left(\sum_{j' \neq j} m_{j'} v_{j'}(x_{j'}) \right)}{\left(\sum_{h=1}^{n_s} m_h v_h(x_h) \right)^3} < 0, \quad (8)$$

and, thus, the objective function is strictly concave.

Given the strict concavity of the objective function, it is clearly suboptimal for the nonprofit in segment j to set $x_j^* = 0$ if there exists an $x_j^* > 0$ which solves segment j 's first-order condition given in equation (7). For each of the k^* participating nonprofits (i.e., $j \in \mathcal{P}$), the first-order condition in equation (7) provides the following necessary condition for equilibrium,

$$m_j v_j(x_j^*) = \left(\sum_{h=1}^{n_s} m_h v_h(x_h^*) \right) - \alpha \left(\sum_{h=1}^{n_s} m_h v_h(x_h^*) \right)^2. \quad (9)$$

Observe that the right-hand side of equation (9) is the same for all $j \in \mathcal{P}$, and recall from equation (1) that $m_j v_j(x_j) = m_j v_j^0 + m_j(1 - v_j^0)x_j$. It, therefore, follows from equation (9) that for each $j \in \mathcal{P}$ the increase in the number of voters $m_j(1 - v_j^0)x_j^*$ is strictly decreasing with respect to $m_j v_j^0$, and thus for $j \in \mathcal{P}$, $m_1(1 - v_1^0)x_1^* \geq \dots \geq m_{k^*}(1 - v_{k^*}^0)x_{k^*}^* > 0$, where k^* is the number of participating nonprofit organizations. The $n_s - k^*$ non-participating nonprofits (i.e., $j \notin \mathcal{P}$) are characterized by $j > k^*$, $x_j^* = 0$, and $m_j v_j(0) = m_j v_j^0$.

Summing across all segments

$$\sum_{j=1}^{n_s} m_j v_j(x_j^*) = k^* \left(\sum_{j=1}^{n_s} m_j v_j(x_j^*) \right) - \alpha k^* \left(\sum_{j=1}^{n_s} m_j v_j(x_j^*) \right)^2 + \left(\sum_{j > k^*} m_j v_j^0 \right), \quad (10)$$

Recalling the definition of the expression \mathcal{V}_k , rearranging equation (10) provides the equilibrium number of voters $\sum_{j=1}^{n_s} m_j v_j(x_j^*)$,

$$\sum_{j=1}^{n_s} m_j v_j(x_j^*) = \mathcal{V}_{k^*} = \frac{\left(k^* - 1\right) + \left[\left(k^* - 1\right)^2 + 4\left(\alpha k^*\right)\left(\sum_{j>k^*} m_j v_j^0\right)\right]^{1/2}}{2\left(\alpha k^*\right)} \quad (11)$$

Recall from equation (1) that $m_j v_j(x_j) = m_j v_j^0 + m_j(1 - v_j^0)x_j$. It follows from equations (9) and (11) that the equilibrium nonpartisan gotv efforts are given by:

$$x_j^* = \begin{cases} \frac{1}{m_j(1-v_j^0)} \left[\mathcal{V}_{k^*} - \alpha \mathcal{V}_{k^*}^2 - m_j v_j^0 \right] & \text{if } j \leq k^* \\ 0 & \text{if } j > k^* \end{cases} \quad (12)$$

To determine which nonprofit organizations choose to participate in nonpartisan gotv efforts recall that the index k^* is such that $m_1(1 - v_1^0)x_1^* \geq \dots \geq m_{k^*}(1 - v_{k^*}^0)x_{k^*}^* > 0$, and $x_j^* = 0$, for $j > k^*$. From the first-order condition given in equation (7), the number of nonprofit organizations that participate in nonpartisan gotv effort k^* is the largest index k such that

$$\mathcal{V}_{(k-1)} - \alpha \mathcal{V}_{(k-1)}^2 > m_k v_k^0 \quad (13)$$

where $\mathcal{V}_{(k-1)}$ is defined as follows

$$\mathcal{V}_{(k-1)} = \frac{\left(k - 2\right) + \left[\left(k - 2\right)^2 + 4\alpha\left(k - 1\right)\left(\sum_{j>(k-1)} m_j v_j^0\right)\right]^{1/2}}{2\alpha\left(k - 1\right)}. \quad (14)$$

This completes the proof of existence. The proof of uniqueness follows along the lines of Matros (2006). \square

Before turning to the formal summary of the nature of the unique equilibrium of the nonpartisan gotv stage (stated in Propositions 2 and 3 below), it is helpful to examine

a simple example which illuminates the main features. Consider an electorate with 100 citizens divided among 4 segments. The cost of nonpartisan gotv efforts is assumed to be $x_j c(m_j, v_j^0) = x_j(0.013)m_j(1 - v_j^0)$. For each segment, Table 1 below provides the number of citizens, the initial voter turnout rate, the unique equilibrium nonpartisan gotv effort, the expected share of the budget, and the initial share of the voters. The segments are arranged in ascending order with respect to the number of citizens, with segment 1 having 20 citizens, segments 2 and 3 having 25 citizens, and segment 4 having 30 citizens. The initial voter turnout rate is 0.5 in segments 1 and 2 and 0.575 in segments 3 and 4. The number of citizens and the initial voter turnout rates are given in columns 2 and 3, respectively, of Table 1.

Segment	m_j	v_j^0	x_j^*	$v_j(x_j^*)$	$m_j E(t_j x_j^*, x_{-j}^*)$	$\frac{m_j v_j^0}{\sum_{j=1}^4 m_j v_j^0}$
1	20	0.500	0.34	0.686	0.232	0.185
2	25	0.500	0.07	0.549	0.232	0.231
3	25	0.575	0	0.575	0.243	0.266
4	30	0.575	0	0.575	0.292	0.318

Table 1: Example

Not all of the segments engage in nonpartisan gotv efforts. From the fourth column of Table 1 we see that in the unique equilibrium in the nonpartisan gotv stage only the nonprofits in segments 1 and 2 participate in nonpartisan gotv efforts (i.e., $x_j^* > 0$ for $j = 1, 2$). As the condition in Theorem 2 states, nonpartisan gotv efforts only occur in those segments in which the product of the size m_j and the initial voter turnout rate v_j^0 is below a threshold. That is, equilibrium nonpartisan gotv efforts only occur in segments that are sufficiently small and disenfranchised.

The fifth column provides the updated voter turnout rates that the political parties use in the campaign stage. Note that the initial share of the voters (reported in the last column of Table 1) provides us with what each segment's expected share of the budget from

the implemented policy would have been if there had not been a nonpartisan gotv stage. Therefore, in comparing the last two columns of Table 1 we see how each segment's expected budget share changes as a result of the nonpartisan gotv stage. In this example, the change in the voter turnout rate (see columns 3 and 5), as a result of the optimal nonpartisan gotv efforts, is increasing in the segment's level of disenfranchisement (i.e., the ex ante abstention rate) and is decreasing in the segment's size. As a result of these changes in the voter turnout rates the political parties place relatively higher weights on the smaller more disenfranchised segments. Comparing the last two columns of Table 1, we see that among the segments in which nonpartisan gotv efforts arise (segments 1 and 2) the change in the expected budget share is higher in the smaller segment (segment 1). As a result of the nonpartisan gotv stage, the change in segment 1's expected budget share is equal to .047 (.232 minus .185) while the change in segment 2's expected budget share is equal to .001 (.232 minus .231). Furthermore, in each of the segments in which nonpartisan gotv efforts do not arise (segments 3 and 4), the expected budget shares decrease.

As formally stated in Propositions 2 and 3, among those segments of the electorate that engage in nonpartisan gotv efforts each segment's increase in the voter turnout rate and the resulting change in the expected budget share are both strictly decreasing with respect to the number of citizens in the segment and the initial voter turnout rate in the segment.

Proposition 2. *In each of the segments in which the nonprofit organizations participate in nonpartisan gotv efforts (i.e., each $j \in \mathcal{P}$ or equivalently $j \leq k^*$), the equilibrium increase in segment j 's voter turnout rate, as a result of the nonpartisan gotv efforts, is strictly decreasing with respect to both segment j 's number of citizens m_j and the initial voter turnout rate v_j^0 .*

From the unique equilibrium target rates $\{x_j^*\}_{j=1}^{n_s}$ given in Theorem 2, it follows that for

each segment $j \in \mathcal{P}$ the increase in segment j 's voter turnout rate is given by:

$$v_j(x_j^*) - v_j^0 = (1 - v_j^0) x_j^* = \frac{\mathcal{V}_{k^*}}{m_j} - \frac{\alpha \mathcal{V}_{k^*}^2}{m_j} - v_j^0 \quad (15)$$

which is clearly decreasing with respect to both segment j 's number of citizens m_j and the initial voter turnout rate v_j^0 . That is, the largest increases in the turnout rates occur in the smallest and most disenfranchised segments.

Given the political parties' optimal strategies in the campaign stage (see Theorem 1), the equilibrium expected budget shares from the implemented policy are increasing with respect to the voter turnout rates (Proposition 1). Thus, to the extent that nonpartisan gotv efforts change the expected turnout rates, nonpartisan gotv efforts have an impact on policy outcomes. As Proposition 2 states, among those segments in which nonpartisan gotv efforts arise, the increase in the voter turnout rate is decreasing with respect to the number of citizens and the initial voter turnout rate. Combining the results from Propositions 1 and 2, we see that among those segments in which nonpartisan gotv efforts arise the change in the expected budget share is also decreasing with respect to the number of citizens and the initial voter turnout rate.

Proposition 3. *In each of the segments in which the nonprofit organizations participate in nonpartisan gotv efforts (i.e., each $j \in \mathcal{P}$ or equivalently $j \leq k^*$), the change in segment j 's equilibrium expected share of the budget from the implemented policy, as a result of the nonpartisan gotv efforts, is strictly decreasing with respect to both segment j 's number of citizens m_j and the initial voter turnout rate v_j^0 .*

In each of the segments in which the nonprofit organizations do not participate in nonpartisan gotv efforts (i.e., each $j \notin \mathcal{P}$ or equivalently $j > k^$), the equilibrium expected budget share from the implemented policy decreases as a result of the nonpartisan gotv efforts in the other segments.*

Given the unique equilibrium expected budget shares, derived in Proposition 1, it follows that for each segment $j \in \mathcal{P}$ the change in segment j 's expected budget share $\Delta m_j E(t_j | \mathbf{x}^*)$ as a result of the gotv activities of nonprofits is

$$\Delta m_j E(t_j | \mathbf{x}^*) = \frac{m_j v_j(x_j^*)}{\sum_{h=1}^{n_s} m_h v_h(x_h^*)} - \frac{m_j v_j^0}{\sum_{h=1}^{n_s} m_h v_h^0} \quad (16)$$

The first part of Proposition 3, then, follows from the equilibrium target rates given in Theorem 2. In particular,

$$\frac{m_j v_j(x_j^*)}{\sum_{h=1}^{n_s} m_h v_h(x_h^*)} = 1 - \alpha \mathcal{V}_K, \quad (17)$$

and, thus, from (16) the change in the expected budget share $\Delta m_j E(t_j | \mathbf{x}^*)$ is decreasing with respect to both segment j 's number of citizens m_j and initial voter turnout rate v_j^0 .

For the second part of Proposition 3, note that for each $j \notin \mathcal{P}$ the change in segment j 's expected budget share $\Delta m_j E(t_j | \mathbf{x}^*)$ as a result of the gotv activities of nonprofits is

$$\Delta m_j E(t_j | \mathbf{x}^*) = \frac{m_j v_j^0}{\sum_{h=1}^{n_s} m_h v_h(x_h^*)} - \frac{m_j v_j^0}{\sum_{h=1}^{n_s} m_h v_h^0}, \quad (18)$$

which is strictly negative if any of the nonprofit organizations engage in nonpartisan gotv efforts.

It is also important to note that just because the nonprofit affiliated with a segment participates in gotv efforts, it is not necessarily the case that the segment's change in the expected budget share is positive. That is, it is possible that among the segments that participate in gotv efforts one or more of the larger and more engaged segments may have a lower expected budget share. However, it is still optimal for the nonprofits in such segments to engage in nonpartisan gotv efforts since not doing so would result in even larger losses from the nonpartisan gotv stage.

4 Conclusion

This paper examines the effects of nonpartisan gotv efforts in a simple multistage game of redistributive politics with voter abstention. For each segment of the electorate in equilibrium the expected transfers from both of the political parties, and hence from the implemented policy, are increasing with respect to the segment's voter turnout rate. In weighing the costs and benefits of nonpartisan gotv efforts, only the nonprofit advocacy groups affiliated with sufficiently small and disenfranchised segments of the electorate engage in nonpartisan gotv efforts. In those segments in which the corresponding nonprofit engages in gotv efforts, the equilibrium increase in the voter turnout rate is decreasing in both the size of the segment and in the initial turnout rate. As a result the smaller more disenfranchised segments gain the most from nonpartisan gotv efforts. These results on the nature and impact of nonpartisan gotv efforts illustrate that even though the political influence activities of nonprofit advocacy organizations may be constrained, these activities influence policy outcomes and are important tools for nonprofit advocacy organizations.

Appendix

This appendix establishes: (a) the existence of joint distributions which satisfy conditions (1) and (2) of Theorem 1 (i.e., form an equilibrium in the campaign stage of the multistage game of redistributive politics with targeted nonpartisan gotv efforts), and (b) the uniqueness of the equilibrium sets of univariate marginal distributions given in condition (2) of Theorem 1. The formal proof of the existence of strategies which satisfy conditions (1) and (2) of Theorem 1 follows lines drawn by Laslier (2002). In this appendix, we only show how the subgame in the campaign stage is isomorphic to the game in that paper.

As mentioned in the description of the model, the subgame in the campaign stage extends Laslier (2002) by allowing for voter abstention. In the case that in each segment the expected

turnout rate (either initially or after the nonpartisan gotv stage) is 1, the two games are equivalent.⁹ In the discussion that follows we show that the equilibria in these two games are related even when the expected turnout rates are not all equal to 1. Recall that within each segment, each party must promise the same transfer to each citizen. Thus, if the equilibrium citizen-level randomization for segment j , given in Theorem 1, is

$$\forall j \in \{1, \dots, n_s\} \quad F_{i,j}^*(t) = \frac{t}{2v_j / \sum_{h=1}^{n_s} v_h m_h} \quad \text{for } t \in \left[0, \frac{2v_j}{\sum_{h=1}^{n_s} v_h m_h}\right],$$

then since there are m_j citizens in segment j the segment-level randomization is given by

$$\forall j \in \{1, \dots, n_s\} \quad F_{i,j}^{s*}(t) = \frac{t}{2v_j m_j / \sum_{h=1}^{n_s} v_h m_h} \quad \text{for } t \in \left[0, \frac{2v_j m_j}{\sum_{h=1}^{n_s} v_h m_h}\right], \quad (19)$$

Letting $\hat{m}_j \equiv m_j v_j(x_j)$, the set of segment-level univariate marginal distributions functions given in equation (19) is identical to that arising in Laslier (2002) and the joint distribution construction given in Lemmas 4-7 of that paper applies directly. Therefore, each party has a strategy that satisfies the restriction on the support given in condition (1) of Theorem 1 — which implies directly that budget-balancing occurs with probability one — and that provides the set of univariate marginal distribution functions stated in condition (2) of Theorem 1.

We now address the uniqueness of the equilibrium sets of univariate marginal distributions given in condition (2) of Theorem 1. The formal proof of this uniqueness follows lines drawn by Roberson (2006). The uniqueness of the equilibrium univariate marginal distributions in the campaign stage follow from the relationship between the subgame in the campaign stage and Roberson (2006). In the discussion that follows we will focus on the segment-level univariate marginal distributions functions given in equation (19). Recall that $\hat{m}_j \equiv m_j v_j(x_j)$. Roberson (2006) examines both the symmetric and asymmetric Colonel Blotto game with homogeneous battlefields and provides a characterization of the equilibrium

⁹Note that in that paper the budget is set to Q , while in this paper the budget has been normalized to 1.

sets of univariate marginal distributions for a range of parameter configurations. By focusing on the segment-level univariate marginal distribution functions and setting $\hat{m}_j \equiv m_j v_j(x_j)$, the subgame in the campaign stage is equivalent to a symmetric Colonel Blotto game with heterogeneous battlefields (i.e., segments of the electorate). In the case of symmetric resources, the proof of the uniqueness of the equilibrium sets of univariate marginal distributions given in Roberson (2006) extends directly to allow for heterogeneous battlefields.

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